Relativistic Entanglement from Relativistic Quantum Mechanics in the Rest-Frame Instant Form of Dynamics.

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Abstract

After a review of the problems induced by the Lorentz signature of Minkowski space-time, like the need of a clock synchronization convention for the definition of 3-space and the complexity of the notion of relativistic center of mass, there is the introduction of a new formulation of relativistic quantum mechanics compatible with the theory of relativistic bound states. In it the zeroth postulate of non-relativistic quantum mechanics is not valid and the physics is described in the rest frame by a Hilbert space containing only relative variables. The non-locality of the Poincare' generators imply a kinematical non-locality and non-separability influencing the theory of relativistic entanglement and not connected with the standard quantum non-locality.

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Atomic physics is an approximation to QED, in which the atoms are described as non-relativistic particles in quantum mechanics (QM) with a coupling to the electro-magnetic field of order $1/c$. For all the applications in which the energies involved do not cross the threshold of pair production, this description with a fixed number of particles is enough. Therefore atomic physics and the theory of entanglement are formulated in the absolute Euclidean 3-space and use Newton absolute time, namely they are formulated in Galilei space-time. The main drawback is that, due to the coupling to the electro-magnetic field there is not a realization of the kinematical Galilei group connecting non-relativistic inertial frames. On the other hand, if we want to arrive at an understanding of relativistic entanglement, we must reformulate the theory in Minkowski space-time with a well defined realization of the kinematical Poincare' group connecting relativistic inertial frames. This would lead to *relativistic atomic physics* as the quantization of a fixed number of classical relativistic charged scalar (or spinning) particles interacting with the classical electro-magnetic field.

In this framework it is possible to take into account the non-local aspects of the Lorentz signature of space-time and of the Poincare' group. Some of the implications are: i) the non-existence of a unique notion of relativistic center of mass; ii) the propagation of rays of light along the light-cone as implied by Maxwell equations. Instead in the non-relativistic theory used for experiments testing entanglement the world-lines of photons do not exist and only their polarization is described by means of a two-dimensional Hilbert space. While for experiments on Earth this is enough, teleportation between the Earth and the Space Station [1] will require a control on the world-lines of light rays. See Refs.[2, 3] for the existing attempt to formulate a relativistic theory of entanglement.

Till now all the attempts to define relativistic QM employ the so-called *zeroth postulate of QM* (see Zurek in Ref.[4]). According to it a composite system of two spatially separated subsystems is described by the *tensor product* of the Hilbert spaces of the subsystems. The notation $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 = \mathcal{H}_{com} \otimes \mathcal{H}_{rel}$ means that the quantum 2-body isolated system can be imagined to be constituted either by the two single particle subsystems with masses m_1 and m_2 or as the tensor product of a decoupled center-of-mass particle of mass $m = m_1 + m_2$ carrying an internal space with an internal relative motion of reduced mass $\mu = m_1\, m_2/m$ [1]. The two descriptions are connected by a unitary transformation and correspond to different choices of bases in $\mathcal{H}$. We will see that the zeroth postulate does not hold at the relativistic level, where the basic conceptual problem is the absence of an intrinsic notion of 3-space without which we cannot formulate a well posed Cauchy problem for Maxwell equations and we loose predictability. This is the problem of clock synchronization, whose clarification came from the attempts to get a consistent description of relativistic bound states.

Moreover the solution must be such that the transition from a simultaneity convention to another one has to be formulated as a gauge transformation (so that physical results are not influenced by the convention, which only modifies the appearances of phenomena).

The standard way out from the problem of 3-space is to choose the Euclidean 3-space of an inertial frame centered on an inertial observer and then use the kinematical Poincare'

[1] The zeroth postulate, i.e. $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$, is based on a notion of *separability* (Einstein's one) independent from the Galilei group, which instead is at the basis of the decomposition $\mathcal{H} = \mathcal{H}_{com} \otimes \mathcal{H}_{rel}$ emphasizing that the center-of-mass momentum is a constant of motion for an isolated system so that one can do the separation of variables in the Schroedinger equation.

group to connect different inertial frames. In the absolute Minkowski space-time (replacing the Newtonian absolute time and absolute Euclidean 3-space) the Euclidean 3-spaces of the inertial frames centered on an inertial observer A are identified by means of Einstein convention for the synchronization of clocks: the inertial observer A sends a ray of light at x^o_i towards the (in general accelerated) observer B; the ray is reflected towards A at a point P of B world-line and then reabsorbed by A at x^o_f; by convention P is synchronous with the mid-point between emission and absorption on A's world-line, i.e. $x^o_P = x^o_i + \frac{1}{2}\,(x^o_f - x^o_i) = \frac{1}{2}\,(x^o_i + x^o_f)$.

However, this is not possible in general relativity (GR), where there is no absolute notion since also space-time becomes dynamical (with the metric structure satisfying Einstein's equations). The equivalence principle implies the absence of global inertial frames: in the restricted class of globally hyperbolic, asymptotically Minkowskian at spatial infinity, space-times the best we can have are global non-inertial frames connected by 4-diffeomorphisms (the gauge group of GR). As a consequence, also in SR we have to face the problem of reformulating physics in non-inertial frames centered on accelerated observers [6] as a first step before facing GR [2].

A metrology-oriented description of non-inertial frames in SR has been done with the *3+1 point of view* and the use of observer-dependent Lorentz scalar radar 4-coordinates [6, 7]. Let us give the world-line $x^\mu(\tau)$ of an arbitrary time-like observer carrying a standard atomic clock: τ is an arbitrary monotonically increasing function of the proper time of this clock. Then we give an admissible 3+1 splitting of Minkowski space-time, namely a nice foliation with space-like instantaneous 3-spaces Σ_τ: it is the mathematical idealization of a protocol for clock synchronization (all the clocks in the points of Σ_τ sign the same time of the atomic clock of the observer). On each 3-space Σ_τ we choose curvilinear 3-coordinates σ^r having the observer as origin. These are the radar 4-coordinates $\sigma^A = (\tau; \sigma^r)$. If $x^\mu \mapsto \sigma^A(x)$ is the coordinate transformation from the Cartesian 4-coordinates x^μ of a reference inertial observer to radar coordinates, its inverse $\sigma^A \mapsto x^\mu = z^\mu(\tau, \sigma^r)$ defines the *embedding* functions $z^\mu(\tau, \sigma^r)$ describing the 3-spaces Σ_τ as embedded 3-manifold into Minkowski space-time. The induced 4-metric on Σ_τ is the following functional of the embedding ${}^4g_{AB}(\tau, \sigma^r) = [z^\mu_A\, \eta_{\mu\nu}\, z^\nu_B](\tau, \sigma^r)$, where $z^\mu_A = \partial\, z^\mu / \partial\, \sigma^A$ and ${}^4\eta_{\mu\nu} = \epsilon\, (+ - - -)$ is the flat metric ($\epsilon = \pm 1$ according to either the particle physics $\epsilon = 1$ or the general relativity $\epsilon = -1$ convention). While the 4-vectors $z^\mu_r(\tau, \sigma^u)$ are tangent to Σ_τ, so that the unit normal $l^\mu(\tau, \sigma^u)$ is proportional to $\epsilon^\mu{}_{\alpha\beta\gamma}\, [z^\alpha_1\, z^\beta_2\, z^\gamma_3](\tau, \sigma^u)$, we have $z^\mu_\tau(\tau, \sigma^r) = [N\, l^\mu + N^r\, z^\mu_r](\tau, \sigma^r)$

[2] Regarding the problem of clock synchronization in presence of gravity near the Earth let us underline the relevance of the ACES mission of ESA [5], programmed for 2013. It will make possible a measurement of the gravitational redshift of the Earth from the two-way link among a microwave clock (PHARAO) on the Space Station and similar clocks on the ground: the proposed microwave link should make possible the control of effects on the scale of 5 picoseconds. This will be a test of post-Newtonian gravity in the framework of Einstein's geometrical view of gravitation: the redshift is a measure of the $1/c^2$ deviation of post-Newtonian null geodesics from Minkowski ones. This is a non-perturbative effect (requiring a re-summation of the whole perturbative expansion) for every quantum field theory, which has to fix the background (and therefore the associated light-cones) to be able to define the quantum Fock space of the theory. As a consequence soon we will also need a formulation of relativistic entanglement in non-inertial frames in presence of post-Newtonian gravity.

($N(\tau, \sigma^r) = \epsilon\, [z^\mu_\tau\, l_\mu](\tau, \sigma^r)$ and $N_r(\tau, \sigma^r) = -\epsilon\, g_{\tau r}(\tau, \sigma^r)$ are the lapse and shift functions).

The foliation is nice and admissible if it satisfies the conditions [3]:
1) $N(\tau, \sigma^r) > 0$ in every point of Σ_τ (the 3-spaces never intersect, avoiding the coordinate singularity of Fermi coordinates);
2) $\epsilon\, {}^4g_{\tau\tau}(\tau, \sigma^r) > 0$, so to avoid the coordinate singularity of the rotating disk, and with the positive-definite 3-metric ${}^3g_{rs}(\tau, \sigma^u) = -\epsilon\, {}^4g_{rs}(\tau, \sigma^u)$ having three positive eigenvalues (these are the Møller conditions [6, 8]);
3) all the 3-spaces Σ_τ must tend to the same space-like hyper-plane at spatial infinity (so that there are always asymptotic inertial observers to be identified with the fixed stars).

In the 3+1 point of view the 4-metric ${}^4g_{AB}(\tau, \vec{\sigma})$ on Σ_τ has the components $\epsilon\, {}^4g_{\tau\tau} = N^2 - N_r\, N^r$, $-\epsilon\, {}^4g_{\tau r} = N_r = {}^3g_{rs}\, N^s$, ${}^3g_{rs} = -\epsilon\, {}^4g_{rs} = \sum_{a=1}^3 {}^3e_{(a)r}\, {}^3e_{(a)s} = \tilde{\phi}^{2/3} \sum_{a=1}^3 e^{2\, \sum_{\bar{b}=1}^2 \gamma_{\bar{b}a}\, R_{\bar{b}}}\, V_{ra}(\theta^i)\, V_{sa}(\theta^i))$, where ${}^3e_{(a)r}(\tau, \sigma^u)$ are cotriads on Σ_τ, $\tilde{\phi}(\tau, \sigma^r) = \sqrt{det\, {}^3g_{rs}(\tau, \sigma^r)}$ is the 3-volume element on Σ_τ, $\lambda_a(\tau, \sigma^r) = [\tilde{\phi}^{1/3}\, e^{\sum_{\bar{b}=1}^2 \gamma_{\bar{b}a}\, R_{\bar{b}}}](\tau, \sigma^r)$ are the positive eigenvalues of the 3-metric ($\gamma_{\bar{a}a}$ are suitable numerical constants) and $V(\theta^i(\tau, \sigma^r))$ are diagonalizing rotation matrices depending on three Euler angles. The components ${}^4g_{AB}$ or the quantities N, N_r, γ, $R_{\bar{a}}$, θ^i, play the role of the *inertial potentials* generating the relativistic apparent forces in the non-inertial frame. It can be shown [6, 8] that the Newtonian inertial potentials are hidden in the functions N, N_r and θ^i. The extrinsic curvature ${}^3K_{rs}(\tau, \sigma^u) = [\frac{1}{2\,N}\, (N_{r|s} + N_{s|r} - \partial_\tau\, {}^3g_{rs})](\tau, \sigma^u)$, describing the *shape* of the instantaneous 3-spaces of the non-inertial frame as embedded 3-manifolds of Minkowski space-time, is a secondary inertial potential functional of the independent inertial potentials ${}^4g_{AB}$.

The description of isolated systems (particles, strings, fields, fluids) admitting a Lagrangian formulation in the non-inertial frames of SR is done by means of *parametrized Minkowski theories* [6, 7]. The matter variables are replaced with new ones knowing the 3-spaces Σ_τ. For instance a Klein-Gordon field $\tilde{\phi}(x)$ will be replaced with $\phi(\tau, \sigma^r) = \tilde{\phi}(z(\tau, \sigma^r))$; the same for every other field. Instead for a relativistic particle with world-line $x^\mu(\tau)$ we must make a choice of its energy sign: then it will be described by 3-coordinates $\eta^r(\tau)$ defined by the intersection of the world-line with Σ_τ: $x^\mu(\tau) = z^\mu(\tau, \eta^r(\tau))$. Differently from all the previous approaches to relativistic mechanics, the dynamical configuration variables are the 3-coordinates $\eta^r_i(\tau)$ and not the world-lines $x^\mu_i(\tau)$ (to rebuild them in an arbitrary frame we need the embedding defining that frame!). Then the matter Lagrangian is coupled to an external gravitational field and the external 4-metric is replaced with the 4-metric $g_{AB}(\tau, \sigma^r)$ of an admissible 3+1 splitting of Minkowski space-time. With this procedure we get a Lagrangian depending on the given matter and on the embedding $z^\mu(\tau, \sigma^r)$, which is invariant under *frame-preserving diffeomorphisms* [9]. As a consequence, there are four first-class constraints (an analogue of the super-Hamiltonian and super-momentum constraints of canonical gravity) implying that the embeddings $z^\mu(\tau, \sigma^r)$ are *gauge variables*, so that all the admissible non-inertial or inertial frames are gauge equivalent, namely physics does *not* depend on the clock synchronization convention and on the choice of the

[3] These conditions imply that global *rigid* rotations are forbidden in relativistic theories. In Ref.[6, 8] there is the expression of the admissible embedding corresponding to a 3+1 splitting of Minkowski space-time with parallel space-like hyper-planes (not equally spaced due to a linear acceleration) carrying differentially rotating 3-coordinates without the coordinate singularity of the rotating disk. It is the first consistent global non-inertial frame of this type.

3-coordinates σ^r: only the appearances of phenomena change by changing the notion of instantaneous 3-space. Even if the gauge group is formed by the frame-preserving diffeomorphisms, the matter energy-momentum tensor allows the determination of the ten conserved Poincare' generators P^μ and $J^{\mu\nu}$ (assumed finite) of every configuration of the system; they are non-local quantities knowing the whole 3-space!.

If we restrict ourselves to inertial frames, we can define the *inertial rest-frame instant form of dynamics for isolated systems* by choosing the 3+1 splitting corresponding to the intrinsic inertial rest frame of the isolated system centered on an inertial observer: the instantaneous 3-spaces, named Wigner 3-space due to the fact that the 3-vectors inside them are Wigner spin-1 3-vectors [7], are orthogonal to the conserved 4-momentum P^μ of the configuration. In Ref.[6] there is the extension to admissible non-inertial rest frames, where P^μ is orthogonal to the asymptotic space-like hyper-planes to which the instantaneous 3-spaces tend at spatial infinity. This non-inertial family of 3+1 splittings is the only one admitted by the asymptotically Minkowskian space-times covered by the canonical gravity formulation of Refs.[10, 11].

In the inertial rest frames we can get the explicit form of the Poincare' generators (in particular of the Lorentz boosts, which, differently from the Galilei ones, are interaction dependent). We can also give the final solution to the old problem of the relativistic extension of the Newtonian center of mass of an isolated system. In its rest frame there are *only* three notions of collective variables, which can be built by using *only* the Poincare' generators (they are *non-local* quantities knowing the whole Σ_τ) [12]: the canonical non-covariant Newton-Wigner center of mass (or center of spin), the non-canonical covariant Fokker-Pryce center of inertia and the non-canonical non-covariant Møller center of energy. All of them tend to the Newtonian center of mass in the non-relativistic limit. See Ref.[7] for the Møller non-covariance world-tube around the Fokker-Pryce 4-vector identified by these collective variables. As shown in Refs.[12–14] these three variables can be expressed as known functions of the rest time τ, of the canonically conjugate Jacobi data (frozen Cauchy data) $\vec{z} = Mc\,\vec{x}_{NW}(0)$ ($\vec{x}_{NW}(\tau)$ is the standard Newton-Wigner 3-position) and $\vec{h} = \vec{P}/Mc$, of the invariant mass $Mc = \sqrt{\epsilon\, P^2}$ of the system and of its rest spin $\vec{\bar{S}}$. As a consequence, every isolated system (i.e. a closed universe) can be visualized as a decoupled non-covariant collective (non-local) pseudo-particle described by the frozen Jacobi data $\vec{z}$, $\vec{h}$ carrying a *pole-dipole structure* (see also Ref.[15]), namely the invariant mass and the rest spin of the system, and with an associated *external* realization of the Poincare' group. The universal breaking of Lorentz covariance is connected to this decoupled non-local collective variable and is irrelevant because all the dynamics of the isolated system leaves inside the Wigner 3-spaces and is Wigner-covariant. In each Wigner 3-space Σ_τ there is a *unfaithful internal* realization of the Poincare' algebra, whose generators are built by using the energy-momentum tensor of the isolated system. While the internal energy and angular momentum are Mc and $\vec{\bar{S}}$ respectively, the internal 3-momentum vanishes: it is the rest frame condition. Also the internal Lorentz boost (whose expression in presence of interactions is given for the first time) vanishes: this condition identifies the covariant non-canonical Fokker-Pryce center of inertia as the natural inertial observer origin of the 3-coordinates σ^r in each Wigner 3-space. As a consequence [16] there are three pairs of second class (interaction-dependent) constraints eliminating the internal 3-center of mass and its conjugate momentum inside the Wigner 3-spaces [16]: this avoids a double counting of the collective variables and allows to re-express the dynamics only in terms of internal Wigner-covariant relative variables. As a consequence, we find that disregarding the unobservable center of mass all the dynamics is described only by relative variables: this is a form of *weak relationism* without the heavy foundational problem of approaches like the one in Ref.[17].

In the case of relativistic particles the reconstruction of their world-lines requires a complex interaction-dependent procedure delineated in Ref.[14]. The final derived world-lines $x_i^\mu(\tau)$ turn out to be non-canonical predictive coordinates, i.e. already at the classical level there is a non-commutative structure implied by Lorentz signature. See Ref.[16] for the comparison with the other formulations of relativistic mechanics developed for the study of the problem of *relativistic bound states*. In Refs.[14, 18] there is the explicit form of the Lorentz boosts for some interacting systems.

In this framework it has been possible to obtain a relativistic formulation of the classical background of atomic physics, considered as an effective theory of positive-energy scalar (or spinning) particles with mutual Coulomb interaction plus the transverse electro-magnetic field of the radiation gauge valid for energies below the threshold of pair production. As shown in Refs.[13] and [16], this has been possible by considering Grassmann-valued electric charges for the particles ($Q_i^2 = 0$, $Q_i\,Q_j = Q_j\,Q_i \neq 0$ for $i \neq j$). It allows a) to make an ultraviolet regularization of Coulomb self-energies; b) to make an infrared regularization eliminating the photon emission; c) to express the Lienard-Wiechert potentials only in terms of the 3-coordinates $\eta_i^r(\tau)$ and the conjugate 3-momenta $\kappa_{ir}(\tau)$ in a way independent from the used (retarded, advanced,..) Green function. All this amount to reformulate the dynamics of the one-photon exchange as a Cauchy problem with well defined potentials. Moreover there is a canonical transformation [16] sending the above system in a transverse radiation field (in- or out-fields) decoupled, in the global rest frame, from Coulomb-dressed particles with a mutual interaction described by the sum of the Coulomb potential plus the Darwin potential. Therefore for the first time we are able to obtain results, previously derived from instantaneous approximations to the Bethe-Salpeter equation for the description of relativistic bound states (see the bibliography of Ref.[13]), starting from the classical theory. Moreover, for the first time, at least at the classical level, we have been able to avoid the Haag theorem according to which the interaction picture does not exist in QFT.

In refs.[21] there is the *multi-temporal quantization* of positive-energy free scalar and spinning particles in a family of global non-inertial frames of Minkowski space-time with space-like hyper-planes as 3-spaces and differentially rotating 3-coordinates defined in Ref.[6]. We take the point of view *not to quantize the inertial effects* (the appearances of phenomena): the embedding $z^\mu(\tau, \sigma^r)$ remains a c-number and we get results compatible with atomic spectra. Instead the problem of the reformulation of particle physics in non-inertial frames is unsolved due to the no-go theorem of Ref.[22] showing the existence of obstructions to the unitary evolution of a massive Klein-Gordon field between two space-like surfaces of Minkowski space-time. This problem has to be reformulated as the search of the class of admissible 3+1 splittings of Minkowski space-time admitting unitary evolution after quantization: this would allow to check whether the hypothesis of non-quantized inertial effects is valid also in field theory (it will be a crucial point for quantum gravity!).

The previous framework allowed to find a new formulation of *relativistic quantum mechanics and entanglement*, which is developed in Ref. [19].

As already said, in Galilei space-time non-relativistic QM, where all the main results about entanglement are formulated, describes a composite system with two (or more) subsystems with a Hilbert space which is the tensor product of the Hilbert spaces of the subsystems: $H = H_1 \otimes H_2$. This type of spatial separability is named *the zeroth postulate* of quantum mechanics. However, when the two subsystems are mutually interacting, one makes a unitary transformation to the tensor product of the Hilbert space H_{com} describing the decoupled Newtonian center of mass of the two subsystems and of the Hilbert space H_{rel} of relative variables: $H = H_1 \otimes H_2 = H_{com} \otimes H_{rel}$. This allows to use the method of separation of variables to split the Schroedinger equation in two equations: one for the free motion of the center of mass and another, containing the interactions, for the relative variables (this equation describes both the bound and scattering states). A final unitary transformation of the Hamilton-Jacobi type allows to replace H_{com} with $H_{com,HJ}$, the Hilbert space in which the decoupled center of mass is frozen and described by non-evolving Jacobi data. Therefore we have $H = H_1 \otimes H_2 = H_{com} \otimes H_{rel} = H_{com,HJ} \otimes H_{rel}$.

While at the non-relativistic level these three descriptions are unitary equivalent, this no more true in relativistic quantum mechanics, due to the previously described problems arising from Lorentz signature like the need of clock synchronization and the non-covariance of the (non-local) canonical center of mass.

In the approach of Ref.[19] we quantize the frozen Jacobi data of the canonical non-covariant decoupled center of mass and the Wigner-covariant relative variables on the Wigner hyper-plane. Since the center of mass is decoupled, its non-covariance is irrelevant: like for the wave function of the universe, who will observe it? Moreover its non-local nature implies that it is not a locally measurable quantity. Finally the use of the static Jacobi data for the external center of mass avoids the causality problems connected with the instantaneous spreading of wave packets (the Hegerfeldt theorem [20]). This viewpoint is in accord with relativistic bound states and relativistic atomic physics

The need of clock synchronization for the definition of the instantaneous 3-spaces and the non-local and non-covariant properties of the decoupled relativistic center of mass, described by the frozen Jacobi data $\vec{z}$ and $\vec{h}$, imply that the only consistent relativistic quantization is based on the Hilbert space $H = H_{com,HJ} \otimes H_{rel}$ ($H_{com,HJ}$ is the Hilbert space of the external center of mass in the Hamilton-Jacobi formulation, while H_{rel} is the Hilbert space of the internal relative variables). The Hilbert space H is not unitarily equivalent to $H_1 \otimes H_2 \otimes ...$, where H_i are the Hilbert spaces of the individual particles. This is due to the fact that already in the non-interacting two-particle case, in the tensor product of two quantum Klein-Gordon fields, $\phi_1(x_1)$ and $\phi_2(x_2)$, most of the states correspond to configurations in Minkowski space-time in which one particle may be present in the absolute future of the other particle, because the two times x_1^o and x_2^o are totally uncorrelated, or in other words there is no notion of instantaneous 3-space (clock synchronization convention). Also the scalar products in the two formulations are completely different as shown in Ref.[23]. In S-matrix theory this problem is eliminated by avoiding the interpolating states at finite (the problem of the Haag theorem) and going the the asymptotic (in the times x_i^o) limit of the free in- and out- states. However in atomic physics we need interpolating states, and not S-matrix, to describe a laser beam resonating in a cavity and intersected by a beam of atoms!

We have also $H \neq H_{com} \otimes H_{rel}$, because if instead of $\vec{z} = Mc\, \vec{x}_{NW}(0)$ we use the evolving (non-local and non-covariant) Newton-Wigner position operator $\vec{x}_{NW}(\tau)$, then we get a violation of relativistic causality because the center-of-mass wave packets spread instantaneously as shown by the Hegerfeldt theorem [20].

Therefore the only consistent Hilbert space is $H = H_{com,HJ} \otimes H_{rel}$, whose non-relativistic limit is the corresponding Newtonian Hilbert space. The main complication is the definition of H_{rel}, because we must take into account the three pairs of (interaction-dependent) second-class constraints eliminating the internal 3-center of mass inside the Wigner 3-spaces. When we are not able to make the elimination at the classical level and formulate the dynamics only in terms of Wigner-covariant relative variables, we have to quantize the particle Wigner-covariant 3-variables η_i^r, κ_{ir} and then to define the physical Hilbert space by adding the quantum version of the constraints a la Gupta-Bleuler.

The quantization defined in Ref.[19] leads to a first formulation of a theory for *relativistic entanglement*. The non validity of the zeroth postulate and the *non-locality* of Poincare' generators imply a *kinematical non-locality* and a *kinematical spatial non-separability* introduced by special relativity, which reduce the relevance of *quantum non-locality* in the study of the foundational problems of quantum mechanics which have to be rephrased in terms of relative variables. Einstein's notion of separability is not valid since in $H = H_{com,HJ} \otimes H_{rel}$ the composite system must be described by means of relative variables in a Wigner 3-space (this is a type of weak form of relationism different from the formulations induced by the Mach principle like in Ref.[17]).

The relativistic formulation of problems like the relevance of decoherence [24] for the selection of preferred robust pointer bases and the emergence of quasi-classical macroscopic objects from quantum constituents will have to be done in terms of relative variables. Moreover, the control of Poincare' kinematics will force to reformulate the experiments connected with Bell inequalities and teleportation in terms of isolated systems containing: a) the observers with their measuring apparatus (Alice and Bob as macroscopic quasi-classical objects); b) the particles of the protocol (but now the ray of light, the "photons" carrying the polarization, move along null geodesics); c) the environment (macroscopic either quantum or quasi-classical object). All these problems disappear as $1/c$ corrections in experiments on Earth, but will be relevant for space physics. A preliminary step for understanding this framework would be to reformulate the non-relativistic theory of entanglement in the Hilbert space $H = H_{com,HJ} \otimes H_{rel}$ and to forget about the decoupled center-of-mass Hilbert space $H_{com,HJ}$.

The final challenge will be a consistent inclusion of the gravitational field, at least at the post-Newtonian level! See Refs.[11] for the use of 3+1 splittings and of radar 4-coordinates in ADM canonical tetrad gravity in globally hyperbolic, asymptotically Minkowskian at spatial infinity, space-times. As shown in Ref.[25], the dynamical nature of space-time implies that each solution of Einstein's equations dynamically selects a preferred 3+1 splitting of the space-time, namely in GR the instantaneous 3-spaces are dynamically determined except for the trace of the extrinsic curvature (the York time ${}^3K(\tau, \sigma^r)$): this inertial gauge variable is the general relativistic remnant of the special relativistic gauge freedom in clock synchronization.

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